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Ultrasonic Characterization of Elastic Anisotropy in Plasma-Sprayed Alumina Coatings

S. Parthasarathi, B.R. Tittmann, K. Sampath, and E.J. Onesto

An ultrasonic, nondestructive contact measurement technique was employed to detect and characterize the elastic anisotropy of a free-standing, plasma-sprayed alumina coating. Following this initial evaluation, a computer-assisted, ultrasonic anisotropic test bed was used to determine the anisotropic elastic stiffness constants of coatings produced by plasma gun currents of 600 and 400 A. The results showed that the plasma-sprayed alumina coatings are transversely isotropic; i.e., isotropic in the spraying direction. These coatings were characterized by five independent elastic stiffness constants. Coatings produced at 600 A plasma gun current showed higher elastic stiffness constants than those produced at 400 A plasma gun current. This increase appeared to be related to a decrease in the porosity content of the coatings produced at the higher plasma gun current.

Nomenclature	
A	anisotropy factor
c	wave velocity
C_{iklm}	elastic stiffness tensor
E	Young's modulus
G	shear modulus
n_i	direction cosines
V_L	longitudinal wave velocity
V_S	shear wave velocity
δ_{im}	Kronecker delta
λ_{im}	Christoffel's tensor
ν	Poisson's ratio
ρ	density

1. Introduction

PROTECTIVE coatings are commonly produced by flame spray, electric-arc spray, or plasma spray processes. These coatings protect the substrate materials from severe service conditions, such as wear, corrosion, and temperature extremes. The performance of coatings depends on their material properties; traditional engineering designs consider only the material properties of the substrate. This is probably due to the limitations of traditional engineering analysis techniques and difficulties associated with the determination of material properties of thin coatings as well as that of the substrate/coating interface. Significant developments in fracture mechanics theory and computer-based, finite-element and finite-difference analysis methods allow engineering analyses in terms of local material properties, if such properties are available. For example, an evaluation of the elastic properties of thermal-sprayed coatings and their isotropic/anisotropic behavior permits engineering analyses involving the effects of service loading conditions on stresses, deflections, etc. within the coating, substrate, and coating/substrate interface. Such analyses can be used in specifying minimum material properties, or in selecting coating materials to preclude potential premature failure of thermal-sprayed coatings. Further, in certain coating/substrate systems, the elastic properties of the coating may show correlations with porosity content and/or cohesive strength. These correlations may be used to obtain desirable coating characteristics, such as reduced porosity content or increased cohesive strength of the coatings, thereby optimizing service performance. In view of such potential, recent reports on thermal-spray coating technologies have identified the need to develop test techniques for measuring various coating characteristics/properties, including elastic properties (Ref 1, 2), to advance the progress of thermal spray technology.

Keywords alumina coatings, anisotropic, elastic modulus, ultrasonic testing

S. Parthasarathi, K. Sampath, and **E.J. Onesto,** Concurrent Technologies Corporation, 1450 Scalp Avenue, Johnstown, PA 15094; **B.R. Tittmann,** Pennsylvania State University, University Park, PA 16802, USA.

Reliable assessment of elastic properties of a coating is often difficult, sometimes because of the small thickness of the coating. Conventional methods employed to determine elastic properties involve stressing the component statically in tension, torsion, or bending and then measuring the strain using strain gages. Such static mechanical testing techniques are well standardized and are widely used to determine the elastic properties of bulk materials. Despite their widespread use, static mechanical test techniques have some disadvantages. For example, static testing for elastic properties is time consuming and expensive; it often requires considerable time and effort in specimen preparation. Another problem with these techniques relates to measuring mechanical displacements with sufficient precision. Further, these conventional techniques do not readily allow determination of the directionality of properties (anisotropy) and local property variations within a bulk material. In addition, most often the static mechanical test techniques cannot be directly applied to evaluate the elastic properties of brittle thermal-spray coatings because these coatings present difficulties in gripping or loading.

Nondestructive techniques for determining elastic properties of coatings do not suffer from such disadvantages. In fact, the success of ultrasonic techniques in determining the elastic properties of various bulk materials, including metals, powder metals, ceramics, and composites, has been demonstrated (Ref 3-5). These investigations showed that the elastic properties are strong functions of density. Ultrasonic techniques have also been used to characterize and quantify microstructural features, such as degree, shape, and orientation of porosity (Ref 6-8). Recently, Xu et al. (Ref 9) demonstrated the application of an ultrasonic guided wave technique based on Lamb and Rayleigh waves to determine the elastic properties of plasma-sprayed Cu-Ni-In coatings on Ti-6Al-4V substrates. Their investigation assumed that the plasma-sprayed coating exhibits isotropic behavior.

Consideration of the plasma-sprayed deposition process shows that a plasma spray (or any other thermal spray) coating cannot exhibit isotropic behavior. For example, consider a typical plasma spray operation wherein spray particles are fed into a plasma arc. Depending on plasma spray conditions and the particle size distribution, the spray particles either experience complete melting, partial melting, or only softening. A high-velocity stream of inert gas or compressed air propels the molten droplets, semisolid or softened spray particles onto a roughened substrate surface where the individual spray particles deposit as splats (with a morphology similar to a pancake) and form a multilayer coating. Although the deposition characteristics of plasma-spray coatings are well documented in the thermal spray literature, most efforts in the mechanical testing for coating property determination (Ref 10-12) and engineering analyses of the mechanical behavior of coatings (Ref 13-16) have traditionally assumed that either thermal-sprayed coatings are isotropic, or have measured a physical property in only one direction. However, researchers have been aware of the anisotropic nature of these coatings (Ref 17, 18). Nakahira et al. (Ref 18) found that the Young's modulus measured in the spraying direction was twice as high as that measured in the perpendicular direction.

This paper reports on the use of ultrasonic techniques to detect elastic anisotropy in plasma-spray coatings, characterize the elastic properties of alumina coatings, and measure their anisotropic elastic stiffness constants.

2. Background

2.1 *Ultrasonic Determination of Elastic Properties*

The application of ultrasonic techniques to determine the elastic properties of materials (including coatings) involves the measurement of velocities of ultrasonic waves. These include longitudinal waves (wherein the particles in the solid material vibrate in the direction of wave propagation) and transverse waves (wherein the particles in the solid material vibrate perpendicular to the direction of wave propagation). These measurements can be used to determine the elastic stiffness constants of both isotropic and anisotropic materials as follows.

An isotropic material is characterized by two independent elastic constants. In such a material, if V_L and V_S are the velocities of longitudinal and shear waves, respectively, then:

$$G = \rho V_S^2 \qquad \text{(Eq 1)}$$

$$\nu = \frac{V_L^2 - 2V_S^2}{2\left(V_L^2 - V_S^2\right)} \qquad \text{(Eq 2)}$$

$$E = 2G(1 + \nu) \qquad \text{(Eq 3)}$$

where G is the shear modulus, E is the Young's modulus, ρ is the density, and ν is the Poisson's ratio. The wave velocities can be determined by launching an ultrasonic pulse into a specimen of known thickness and by measuring the time taken for the pulse to travel a known distance. Thus, based on thickness, density, and wave velocity measurements, G, ν, and E can be determined for an isotropic material.

In the case of an anisotropic material, Christoffel's equation, which relates wave propagation directions and particle displacement directions with the elastic stiffness constants, can be used to characterize the type of anisotropy (Ref 19). A transversely isotropic material is characterized by 5 independent elastic stiffness constants. An orthotropic material exhibits 9 independent elastic stiffness constants. A material exhibiting the most general case of anisotropy is characterized by 21 independent elastic stiffness constants. Christoffel's equation is:

$$(\lambda_{im} - \rho c^2 \delta_{im})U_m = 0 \qquad \text{(Eq 4)}$$

where c is the wave velocity, ρ is the material density, U_m is the particle displacement vector, and δ_{im} is the Kronecker delta. Christoffel's tensor, λ_{im}, is given as:

$$\lambda_{im} = C_{iklm} n_k n_l \qquad \text{(Eq 5)}$$

where C_{iklm} denote elastic stiffness tensors, and n_i denote the direction cosines of the wave propagation direction. Equation 5 is represented in Einstein convention (repeated suffixes indicate summation); for example, a representative element of the tensor λ_{23} is obtained by the following summation:

$$\lambda_{23} = \sum_{k=1}^{3} \sum_{l=1}^{3} C_{2kl3} n_k n_l \qquad \text{(Eq 6)}$$

Thus, based on measurements of the velocities of ultrasonic waves propagating along different directions, one can evaluate the number of elastic stiffness constants of a material and thereby determine the type and extent of anisotropy.

Ultrasonic excitation of a surface of an infinite anisotropic solid propagates three types of waves into the material. The faster longitudinal wave is followed by two transverse waves. The particle displacement of three waves are mutually perpendicular. For longitudinal wave propagation in a perpendicular direction, if the particle displacements of the wave are in the direction of propagation of the wave, the mode is called a pure mode, and the wave is referred to as a pure longitudinal wave. Otherwise the wave propagation mode is called a quasi-mode, and the wave is referred to as a quasi-longitudinal wave. Similarly, for transverse wave propagation, if the particle displacements are perpendicular to the propagation direction, the wave is called a pure transverse wave. Otherwise the transverse wave is referred to as a quasi-transverse wave.

Following measurement of the velocities of these quasi-longitudinal and quasi-transverse waves at various incident angles, the elastic stiffness constants are determined using Christoffel's equation (Ref 20, 21). For example, based on ultrasonic contact measurements of velocities of waves with different propagation and polarization directions, Christoffel's equation is solved to obtain expressions for wave velocities as a function of elastic stiffness constants for wave propagation along directions in which wave velocity measurements were performed. The elastic stiffness constants then can be extracted from these equations. The section "Appendix: Extraction of Elastic Stiffness Constants" describes such a procedure to extract the stiffness constants of a transversely isotropic material. However, based on ultrasonic contact measurements made in three orthogonal directions (i.e., normal and two transverse directions) using normal incidence, four of the five elastic stiffness constants of a transversely isotropic material—C_{11}, C_{22}, C_{55}, and C_{23} (but not C_{12})—can be determined. The entire anisotropic elastic stiffness tensor of a transversely isotropic material is determined by using the ultrasonic anisotropy test bed (available at Pennsylvania State University) to perform the ultrasonic measurements over a range of incident angles, wave propagation, and polarization directions. An iterative procedure can be employed to recover the elastic stiffness constants by measuring the velocities of these quasi-longitudinal and quasi-transverse waves at various incident angles.

3. Experimental

3.1 *Test Specimen Preparation*

Several, free-standing alumina coatings (Metco 130 by Sulzer/Metco, 1101 Prospect Ave., Westburg, NY 11590, containing 87% Al_2O_3 and 13% TiO_2) were produced, initially on rectangular steel substrates (type AISI 1020 low-carbon steel) using a Sulzer-Plasma Technik A3000S atmosphere powder plasma spray system. Table 1 shows the set points for specific plasma-spray process variables. The as-received surfaces of the steel substrates were cleaned with a degreasing agent and not subjected to a grit-blasting operation to achieve poor or limited adhesion. This type of substrate surface preparation enables easy removal of the coatings from the substrates. Subsequently, the free-standing, plasma-sprayed alumina coatings were sectioned into rectangular prisms (typically 12 by 12 by 7 mm) using a diamond saw. These rectangular prism sections were used in the contact ultrasonic experiments.

Table 1 Set points for plasma-spray process variables

Process variable	Set point
Plasma gas flow, L/min	45
Carrier gas flow, L/min	3.5
Voltage, V	57
Current, A	600
Spray distance, mm	250

Note: The plasma gas flow rate of 45 L/min is made up of an argon flow of 35 L/min and a hydrogen flow of 10 L/min (argon/hydrogen ratio = 3.5). The carrier gas was argon.

The anisotropic elastic stiffness constants were determined using an anisotropic test bed; two additional sets of free-standing test specimens were produced. The first set was produced with the same set of plasma-spray process parameters as shown in Table 1. A second set of coatings was produced with a 400 A gun current while all other parameters were maintained as shown in Table 1.

3.2 *Thickness and Density Measurements*

The thickness and density of each specimen were determined following test specimen preparation. The thickness measurements were performed using a micrometer. The density measurements were performed using an Archimedes oil-immersion procedure recommended by the Metal Powder Industries Federation (MPIF) Standard 42 for porous materials (Ref 22). This technique, having been developed for porous materials, accounts for the presence of connected surface porosity.

3.3 *Ultrasonic Contact Measurements*

Subsequently, the ultrasonic contact measurements at normal incidence were performed using the rectangular prismatic sections (typically 12 by 12 by 7 mm) of the plasma-sprayed alumina coating. Figure 1 shows a schematic diagram of a plasma-spray coating test specimen to illustrate wave propagation and particle displacement (polarization) directions for normal incidence. Ultrasonic measurements were performed using matched transducers at a center frequency of 5 MHz. A computer-controlled MATEC Instruments SR9000 DSP ultrasonic system (Matec Instruments, 56 Hudson St., North Burough, MA 01532) was used to measure the velocities of longitudinal and shear waves propagating along different directions in the specimen. Following wave velocity measurements, Christoffel's equations were used to obtain expressions for wave velocities as a function of elastic stiffness constants for wave propagation along directions in which the wave velocity measurements were performed, and the elastic stiffness constants were determined. (This procedure is described in the appendix).

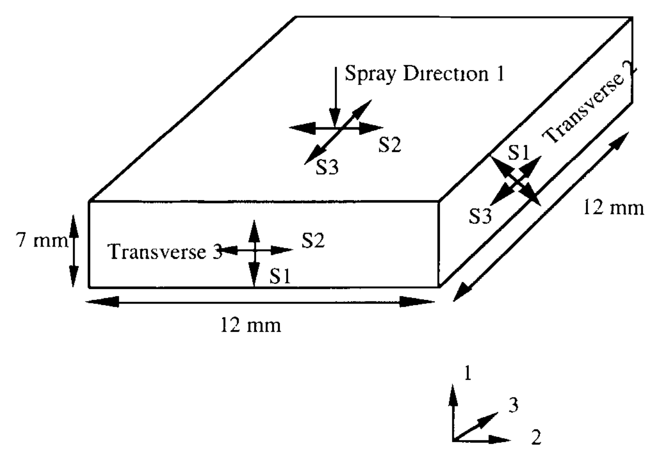

Fig. 1 Schematic diagram of test specimen, illustrating wave incidence, propagation, and polarization directions

3.4 *Measurements Using Immersion Ultrasonics in an Anisotropic Test Bed*

Figure 2 schematically illustrates the ultrasonic anisotropy test bed used to determine the elastic stiffness constants of the free-standing, plasma-spray coatings. The test setup consists of a computer-assisted ultrasonic apparatus to measure the velocities of quasi-longitudinal and quasi-transverse waves propagating at various angles. This apparatus includes a Panametrics 6400 pulser/receiver to generate and receive ultrasound, a Klinger stepper motor controller (Klinger, Irvine, CA 92714) to control both the rotary motion of the goniometer and the linear motion of the slide, a LeCroy oscilloscope (LeCroy Corp., 700 Chestnut Ridge Rd., Chesnut Ridge, NY 10977) for data acquisition, and an Apple Macintosh computer for data analysis and extraction of elastic constants. The stepper motor has 0.01° rotary resolution and 1 mm linear resolution.

Figure 3 shows the specimen fixturing used with the experimental measurements. Initially, a test specimen was mounted on an aluminum plate using an epoxy adhesive. This plate then was placed in a water tank and rotated through the use of a goniometer to generate various incident angles. The fixture provided structural stability to the test specimen during rotary manipulation. The ultrasonic measurements were performed in a through-transmission setup using matched transducers at a center frequency of 5 MHz. The incidence angle was varied in 3° steps from 0° to 45° in a plane. This resulted in approximately 30 measurements per plane (including both longitudinal and transverse wave measurements). The wave velocity measurements were made at 0°, 45°, and 90° planes. This was accomplished by changing the axis of rotation of the test specimen, as shown in Fig. 3. Measurements were performed on two specimens obtained from each process variable set. The thickness of these specimens ranged from 1.7 to 2.5 mm.

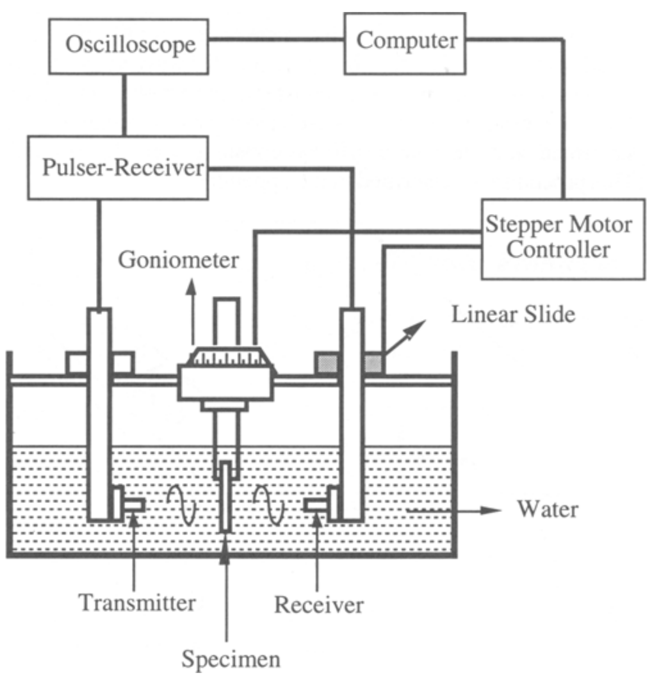

Fig. 2 Schematic of anisotropic test bed

The elastic stiffness constants were extracted using a least squares optimization process (Ref 21) following the wave velocity measurements. Compared to the contact ultrasonic measurement method, which assumes the coating is transversely isotropic, this technique initially assumes a general case of orthotropy (three planes of elastic symmetry) and progressively evaluates different types of anisotropy until a convergence is established.

4. Results and Discussion

4.1 *Thickness and Density Measurements*

Table 2 shows the results of the thickness and density measurements. The precision of the density measurements was typically better than 1%. These results indicate a marginal reduction in coating density with decreasing plasma gun current.

4.2 *Contact Measurements*

Table 3 shows the measurements of ultrasonic contact wave velocity at 5 MHz. V_L refers to a longitudinal wave; V_{S1}, V_{S2}, and V_{S3} refer to shear waves whose particle displacements (polarizations) and propagation directions are shown in Fig. 1.

The wave velocity measurements show the following. (a) The longitudinal and shear wave velocities are higher in the spray direction compared to the transverse directions, indicating the presence of anisotropy. (b) The shear waves polarized in perpendicular directions for propagation along the spray direction show minimal difference in velocities. (c) Both longitudinal and shear wave velocities measured in the two transverse directions show minimal difference. (d) The shear waves propagating in the transverse directions, but polarized in perpendicular directions, show a large difference in wave velocity.

These observations indicate that the plasma spray coating is most likely transversely isotropic; that is, isotropic in the plane of the coating. Such a quasi-isotropic behavior is similar to that observed in unidirectionally reinforced composites (Ref 23) and unidirectionally compacted particulate materials (Ref 4-5). As-

Table 2 Results of thickness and density measurements

No.	Current, A	Thickness, mm	Density, g/cm^3
1	600	7.066	3.423
2	600	2.754	3.409
3	600	2.274	3.402
4	400	1.562	3.372
5	400	1.559	3.382

Note: The calculated theoretical density is about 3.9 g/cm^3.

Table 3 Results of ultrasonic wave velocity measurements

Propagation direction	Velocity, m/s V_L	V_{S1}	V_{S2}	V_{S3}
Spray	6090	...	3575	3566
Transverse 2	4884	3567	...	3272
Transverse 3	4850	3556	...	3267

suming that the plasma spray coating is transversely isotropic, we used Christoffel's equations to obtain expressions for wave velocities as a function of elastic stiffness constants for wave propagation along directions in which wave velocity measurements were performed. Four elastic stiffness constants (C_{11}, C_{22}, C_{55}, and C_{23}) were extracted from these expressions. The elastic stiffness constants of the two sets of coatings were determined using the procedure outlined in the appendix following the wave velocity measurements performed using the anisotropic test bed. In the calculations, the average wave velocities in the two transverse directions was used. Table 4 (specimen No. 1) shows the calculated values of these constants (in GPa).

4.3 *Anisotropic Test Bed*

Table 4 shows the reconstructed elastic stiffness constants (in GPa) for the two sets of coatings (specimen No. 2 to 5). Minor variations observed in the calculated elastic stiffness constants from specimen to specimen arise from errors in the density and velocity measurements as well as spatial variations in properties within the specimen. The calculated values of C_{11}, C_{55}, and C_{22} obtained using the test bed were approximately equal to the values calculated from the preliminary normal incidence contact ultrasonic measurements (specimen No. 1). However, a high value of C_{23} was obtained from the contact measurements compared to that obtained using the anisotropy test bed. This difference in test results is attributed to errors in the contact measurements arising from lack of coupling uniformity and test specimen parallelism.

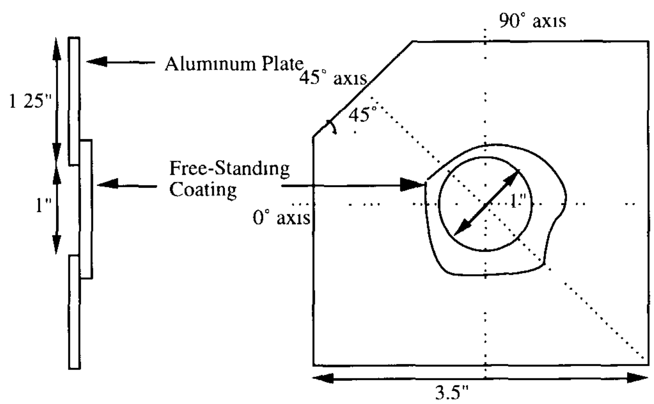

Fig. 3 Plan and elevation views of free-standing, plasma-sprayed alumina coating utilized for ultrasonic anisotropic test bed measurements

An examination of the elastic stiffness constants determined from the anisotropy test bed measurements showed that $C_{12} \approx C_{13}$; $C_{22} \approx C_{33}$; $C_{55} \approx C_{66}$; and $C_{44} \approx (C_{22} - C_{23})/2$. These relationships among the elastic stiffness constants indicated that the free-standing, plasma-sprayed alumina coatings are most likely transversely isotropic; that is, the coatings is isotropic about the spraying (1) axis. This behavior is consistent with the hypothesis on the evolution of anisotropy in plasma-sprayed coatings, based on an analysis of the deposition process.

Table 4 also shows that the elastic stiffness constants of the coatings produced at 600 A plasma gun current were about 20% higher than that of the coatings produced at 400 A. Wave velocity measurements performed using the anisotropic test bed also showed that the longitudinal wave and shear wave velocities in the test specimens typically increase with increasing gun current; this could be attributed to increased density. In fact, Steeper et al. (Ref 25), in employing a Taguchi procedure to evaluate the influence of various plasma-spray process parameters on the porosity content of coating, showed that increases in the plasma gun current decrease the degree of porosity. Thus, based on measurement of wave velocities and determination of elastic stiffness constants, one could estimate the porosity content of a plasma-spray coating and identify possible methods to optimize plasma-spray process parameters.

The extent of anisotropy in anisotropic materials can be characterized by an anisotropy factor (Ref 19), which is defined as:

$$\text{Anisotropy factor} = \frac{2C_{44}}{C_{11} - C_{12}} \tag{Eq 6}$$

This constant represents the degree of anisotropy and is equal to 1 for an isotropic material. The calculated values of the elastic stiffness constants determined from the anisotropy test bed measurements show that the anisotropy constant varies between 0.70 and 0.78.

5. Conclusions

This work demonstrates the use of ultrasonic techniques to characterize elastic anisotropy and to determine elastic stiffness constants of free-standing, plasma-sprayed alumina coatings. The results indicate that the plasma-sprayed alumina coatings

Table 4 Elastic stiffness constants (in GPa) and anistropy factor

No.	Current, A	C_{11}	C_{22}	C_{33}	C_{44}	C_{55}	C_{66}	C_{12}	C_{13}	C_{23}	A
Contact ultrasonic measurements											
1	600	127.36	81.34	...	43.76	...	...	...	...	44.63	...
Anisotropy test bed measurements											
2	600	129.66	91.24	83.46	35.31	34.77	36.79	32.32	29.46	30.02	0.73
3	600	134.90	100.85	98.57	38.21	38.59	46.15	33.96	33.03	21.31	0.76
4	400	109.82	85.08	83.98	29.42	29.53	33.76	26.07	27.82	14.90	0.70
5	400	102.88	90.58	87.11	28.78	31.54	23.99	29.16	26.27	19.52	0.78

A, anisotropy factor = $(2 \times C_{44})/(C_{11} - C_{12})$; E, bulk alumina = 400 GPa (Ref 25) exhibit elastic anisotropy. When an automated ultrasonic anisotropic test bed was used for wave velocity measurements, the calculated elastic stiffness constants showed that the coatings are transversely isotropic. This behavior is consistent with the plasma-spray process deposition characteristics. The elastic stiffness constants were dependent on the porosity content of plasma-sprayed coatings. Fracture, fatigue, wear, and other computer-based techniques of engineering analyses of coatings could use these anisotropic elastic stiffness constants to better evaluate the mechanical behavior of the coatings in service.

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Appendix: Extraction of Elastic Stiffness Constants

This appendix describes a procedure for extracting elastic stiffness constants of a transversely isotropic material from normal incidence contact ultrasonic velocity measurements in spraying and transverse directions. A transversely isotropic material has five independent elastic constants; using condensed notation, these are C_{11}, C_{12}, C_{22}, C_{23}, C_{55}, where the subscript 1 denotes the axis about which it is isotropic. The stiffness tensor for a transversely isotropic (five independent elastic constants) material is:

$$\begin{bmatrix} C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{12} & C_{23} & C_{22} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{C_{22}-C_{23}}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{55} \end{bmatrix}$$

l_{im} for transversely isotropic material, as a function of direction cosines (n_1, n_2, and n_3) of the propagation direction, can be obtained from the expression $l_{im} = C_{iklm} n_k n_l$:

$$l_{11} = C_{11} n_1^2 + C_{55} n_2^2 + C_{55} n_3^2$$

$$l_{22} = C_{55} n_1^2 C_{22} n_2^2 + \frac{C_{22}-C_{23}}{2} n_3^2$$

$$l_{33} = C_{55} n_1^2 + \frac{C_{22}-C_{23}}{2} n_2^2 + C_{22} n_3^2$$

$$l_{12} = (C_{55} + C_{12}) n_1 n_2$$

$$l_{13} = (C_{55} + C_{12}) n_1 n_3$$

$$l_{23} = \frac{C_{22}+C_{23}}{2} n_2 n_3 \qquad \text{(Eq 7)}$$

For propagation along the 1-axis ($[n_1 n_2 n_3] = [100]$) and from the above equation set, l_{im} is calculated as:

$$\begin{bmatrix} C_{11} & 0 & 0 \\ 0 & C_{55} & 0 \\ 0 & 0 & C_{55} \end{bmatrix}$$

This results in the following eigenvalue problem.

$$\begin{bmatrix} C_{11} & 0 & 0 \\ 0 & C_{55} & 0 \\ 0 & 0 & C_{55} \end{bmatrix} - \rho c^2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} U_m = 0$$

Solving the eigenvalue problem, for propagation along the spraying direction (1-axis), the following expressions for the velocities of the three waves can be obtained:

$$c_1^2 = \frac{C_{11}}{\rho}$$

$$c_2^2 = \frac{C_{55}}{\rho}$$

$$c_3^2 = \frac{C_{55}}{\rho} \qquad \text{(Eq 8)}$$

Polarization (direction of particle motion) of each wave can be determined by substituting the appropriate expression from Eq 8 into the following matrix:

$$\begin{aligned} a_1 l_{11} + a_2 l_{12} + a_3 l_{13} &= a_1 r c^2 \\ a_1 l_{12} + a_2 l_{22} + a_3 l_{23} &= a_2 r c^2 \\ a_1 l_{31} + a_2 l_{23} + a_3 l_{33} &= a_3 r c^2 \end{aligned} \qquad \text{(Eq 9)}$$

For c_1, the polarizations $[a_1 a_2 a_3]$ are found to be [100]. Hence c_1 is a longitudinal wave. When the same procedure is followed, c_2 and c_3 are found to be shear waves. Since the material is assumed to be transversely isotropic, the polarization direction of the shear waves is immaterial. Solving Eq 8, C_{11} and C_{55} can be calculated.

Similarly, for propagation along the transverse direction (3 axis), the following expressions for wave velocities can be obtained:

$$c_1^2 = \frac{C_{22}}{\rho}$$

$$c_2^2 = \frac{(C_{22} - C_{23})}{\rho}$$

$$c_3^2 = \frac{C_{55}}{\rho} \quad \text{(Eq 10)}$$

c_1 is a longitudinal wave, and c_2 and c_3 are shear waves polarized in the 2 and 1 directions, respectively. Thus, C_{22} and C_{23} can be calculated.

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